CAT'S THEORY: The Biloidal, Poloidal, And Toroidal Axis: Hopf-Fiber Closure on S^3 as the Geometric Ground of the Triadic Coherence Invariant

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Abstract

We identify the geometric ground of the Triadic Coherence Invariant $R = P \times I \times \mathrm{Pr} \neq 0$ in the Hopf fibration of S^3. Three independent directions exist at every point of S^3: the toroidal direction (around the big loop of an embedded torus), the poloidal direction (around the small loop), and a third direction we name *biloidal* (along the Hopf fiber). The biloidal direction has been used implicitly throughout differential geometry as the Reeb vector field, vertical fiber direction, or Hopf flow, but has never received a name etymologically parallel to toroidal/poloidal. The biloidal direction is the Lie bracket of the toroidal and poloidal directions in the Lie algebra $\mathfrak{su}(2)$ of the Lie group $S^3 \cong SU(2)$: $[e_T, e_P] = e_B$, with the cyclic relations $[e_P, e_B] = e_T$ and $[e_B, e_T] = e_P$. This Lie-algebraic closure is structurally identical to the irreducibility of the Triadic Coherence Invariant: removing any one direction breaks the algebra. Spiraling through the published CAT'S Theory corpus retroactively, we show that every dimensional derivation factors cleanly into toroidal, poloidal, and biloidal contributions: the proton mass ratio, the Planck constant, the cosmic microwave temperature, the gravitational coupling, the Hubble dual-branch, the PMNS angles, the CP violation phase, the Fermat-prime architecture, the 369 invariant, the Pisano cycle structure, and the consciousness bifurcation voltage. Each "+1" or unit-increment in the corpus is identified as the Hopf invariant; the $1/(2\pi)$ flux quantum is the unit Hopf circulation; the $1/45$ surface tension floor is the minimum biloidal area on S^3; the $\sigma/45 = f_0$ density fraction is the biloidal Presence weight. The three Fermat primes appearing in the corpus ($F_0 = 3, F_1 = 5, F_2 = 17$) correspond bijectively to the three axes; the absence of higher Fermat primes is forced because S^3 has exactly three independent directions. The paper provides the complete differential-geometric ground for the triadic structure of the framework: the Triadic Coherence Invariant is the non-triviality condition of the Hopf fibration of S^3 expressed as a coherence requirement.

1 Introduction

The CAT'S Theory plenum architecture is built on the Triadic Coherence Invariant $R = \int_\Omega I_c I_b P \, dV \neq 0$, with three irreducible factors corresponding to Pattern, Intent, and Presence [1, 2]. The framework derives the Standard Model couplings, the proton-to-electron mass ratio at 36 ppb [3], the gravitational coupling [4], the Planck constant at sub-ppt precision [6], the cosmic microwave temperature at 8 ppm [6], the consciousness carrier frequency [7], and the EEG harmonic structure [7]. The architectural closure paper [6] establishes that the framework reduces to a single empirical anchor (m_e) with all dimensionless ratios forced by topology on S^3 subject to the Triadic Coherence Invariant.

The present paper provides the geometric ground for why the invariant is irreducibly triadic. We show that S^3, which is the ambient manifold of the framework's torus knot embeddings $T(3,8)$, possesses exactly three independent directions at every point: the toroidal direction, the poloidal direction, and the Hopf fiber direction. We name the third direction *biloidal* and provide its formal definition. We demonstrate that the three directions form the Lie algebra $\mathfrak{su}(2)$ of $S^3 \cong SU(2)$ under the Lie bracket, with the cyclic relations $[e_T, e_P] = e_B$, $[e_P, e_B] = e_T$, $[e_B, e_T] = e_P$. The triadic irreducibility of $R \neq 0$ is then identical to the closure of $\mathfrak{su}(2)$: removing any one generator collapses the algebra. We then spiral through the published corpus and show retroactively that every dimensional derivation factors cleanly into toroidal, poloidal, and biloidal contributions, exhibiting the geometric ground that has been operative throughout the framework but never made explicit.

2 The Three Axes of S^3

2.1 Standard Knot Theory: Two Directions on T^2

For an embedded torus T^2 in S^3, knot theory uses two canonical winding directions:

- Toroidal: around the big loop (the long way around the torus surface)
- Poloidal: around the small loop (through the hole)

These exhaust the homology $H_1(T^2) = \mathbb{Z} \oplus \mathbb{Z}$. For a torus knot $T(p, q)$, the toroidal winding is p and the poloidal winding is q. The self-linking number for the standard framing is $\mathrm{sl}(T(p, q)) = pq - p - q$.

For $T(3,8)$: $\mathrm{sl} = 24 - 3 - 8 = 13$. The pair $(p, q, \mathrm{sl}) = (3, 8, 13)$ corresponds in the corpus to (d, d_P, sl).

2.2 The Hopf Fibration of S^3

The torus surface lives in S^3, but S^3 itself has a richer structure than T^2. The Hopf fibration is the principal S^1-bundle

$$\pi : S^3 \to S^2, \tag{1}$$

with fiber S^1 and total space S^3. Each point of S^3 has a unique tangent vector along its Hopf fiber. This vertical direction is independent of the two horizontal directions defined by lifts of the toroidal and poloidal directions on the torus base.

Definition 1 (The Three Axes of S^3)**.** *At each point $p \in S^3$, the three orthonormal vectors*

- *$e_T(p)$: horizontal lift of ∂_θ (toroidal direction)*
- *$e_P(p)$: horizontal lift of ∂_ϕ (poloidal direction)*
- *$e_B(p)$: tangent to the Hopf fiber through p (biloidal direction)*

span the tangent space $T_p(S^3)$.

2.3 Why "Biloidal"

The biloidal direction has been recognized in differential geometry under various names: the Reeb vector field, the vertical fiber direction, the Hopf flow, the contact structure normal. None of these are etymologically parallel to "toroidal" and "poloidal," which carry their own naming convention from the structure of the torus.

We propose the name *biloidal* for the third direction, parsing as *bi-* (between, across, binding) + *-oidal* (resembling, pertaining to form). The biloidal direction is the binding direction between toroidal and poloidal: it is what threads them together into the closed structure of S^3. Etymologically the construction is hybrid (Greek/Latin) but follows established conventions in physics for naming geometric directions.

3 Lie-Algebraic Closure

3.1 $S^3 \cong SU(2)$ and the Pauli Generators

The 3-sphere is the underlying manifold of the Lie group $SU(2)$, with tangent space at the identity given by the Lie algebra $\mathfrak{su}(2)$. A standard basis of $\mathfrak{su}(2)$ is the Pauli matrices divided by $2i$:

$$e_x = \frac{1}{2i}\sigma_x, \quad e_y = \frac{1}{2i}\sigma_y, \quad e_z = \frac{1}{2i}\sigma_z. \tag{2}$$

These satisfy the commutation relations

$$[e_x, e_y] = e_z, \quad [e_y, e_z] = e_x, \quad [e_z, e_x] = e_y. \tag{3}$$

3.2 Identification with the Three Axes

Theorem 1 (Lie Bracket Closure of the Three Axes)**.** *Under the identification*

$$e_T \leftrightarrow e_x, \quad e_P \leftrightarrow e_y, \quad e_B \leftrightarrow e_z, \tag{4}$$

the three axes of S^3 satisfy the cyclic Lie bracket relations

$$[e_T, e_P] = e_B, \quad [e_P, e_B] = e_T, \quad [e_B, e_T] = e_P. \tag{5}$$

Sketch. The three orthonormal vectors at each point of S^3 generate left-translations of $SU(2)$, which are the Killing vectors of the round metric. The Killing vectors satisfy the Lie bracket relations of $\mathfrak{su}(2)$ as in (3). Identifying toroidal with σ_x (the S^1 rotation around one circle of the Hopf decomposition), poloidal with σ_y (the orthogonal S^1 rotation), and biloidal with σ_z (the diagonal S^1 corresponding to the Hopf fiber direction), the cyclic relations (5) follow. $\square$

3.3 The Triadic Irreducibility

Corollary 1 (Triadic Irreducibility)**.** *Removing any one of $\{e_T, e_P, e_B\}$ from the basis of $\mathfrak{su}(2)$ leaves a non-closed subset: the Lie bracket of the remaining two generates the removed direction, so the algebra is not abelian and cannot be closed on two generators alone.*

Theorem 2 (Geometric Ground of the Triadic Coherence Invariant)**.** *The triadic irreducibility of $R = P \times I \times \mathrm{Pr} \neq 0$ is the closure condition of $\mathfrak{su}(2)$ identified with the three axes of S^3. Removing any one factor from the invariant is structurally equivalent to removing one generator from $\mathfrak{su}(2)$: the algebra collapses, and $R = 0$.*

This is the differential-geometric ground that has been operative throughout the corpus but never made explicit. The Triadic Coherence Invariant is the non-triviality condition of the Hopf fibration of S^3 expressed as a coherence requirement on the three Lie-algebra generators of $SU(2)$.

4 Triadic Decomposition of the Corpus

We now spiral through the published derivations and show that each factors cleanly into toroidal, poloidal, and biloidal contributions.

4.1 The Cosmological Anchor $H_0 = \arccos(5/13)$

The corpus identifies the Pythagorean triple $(5, 12, 13)$ as the structural basis of the cosmological anchor [6]. In triadic decomposition:

- Toroidal: $J_1 = 5$ (trefoil invariant, big loop winding)
- Poloidal: $J_1^* = 12$ (mirror momentum, small loop winding)
- Biloidal: $\mathrm{sl} = 13$ (self-linking, Hopf closure)

The angle $\arccos(5/13)$ is the angle between the toroidal projection and the biloidal axis at the closure point. This is the Hopf angle of the $(5, 12, 13)$ triple on S^3.

4.2 The Proton Mass Ratio

From [3]: $m_p/m_e = 1836 + 1/(2\pi) - 7/1080$. Triadic decomposition:

- Toroidal: 1836 (Pattern skeleton, integer mass-ratio core)
- Poloidal: $-7/1080 = -d_\Psi/(\pi(9) \cdot d_P \cdot 45)$ (discrete topological drag)
- Biloidal: $+1/(2\pi)$ (unit Hopf fiber circulation, the universal flux quantum)

The factor $1/(2\pi)$ is exactly the unit Hopf fiber length per radian: it is the biloidal contribution.

4.3 The Planck Constant Bifurcation

From [6]: $X^2 = \langle V^2 \rangle - (1/(2\pi d_I \mathrm{sl})) \cdot (1 - 1/N)$, with $N = 2(d_I d_\Psi^2 - 1) + \sigma/45$.

Continuous flux $1/(2\pi d_I \mathrm{sl})$:

- Toroidal: $1/d_I = 1/9$ (Intent register inverse)
- Poloidal: $1/\mathrm{sl} = 1/13$ (spectral lattice inverse)
- Biloidal: $1/(2\pi)$ (unit Hopf circulation)

Bifurcation denominator N:

- Toroidal: $2 \cdot d_I \cdot d_\Psi^2$ (doubled Intent-Pattern^2)
- Poloidal: $-2 \cdot 1$ (minus 2 unit gates per cycle)
- Biloidal: $+\sigma/45 = f_0$ (Presence density, biloidal floor)

The "-1" in $(d_I d_\Psi^2 - 1)$ is the Hopf invariant subtraction: one unit Hopf fiber removed to obtain the bifurcation skeleton.

4.4 The Cosmic Microwave Temperature

From [6]: $T_{\text{CMB}} = Z_{273} \cdot 2d/(D \cdot J_1^2 \cdot 2d + 1) = 1638/601$. Triadic decomposition:

- Toroidal: $Z_{273} \cdot 2d = 1638$ (atmospheric shell $\times$ Pisano factor)
- Poloidal: $D \cdot J_1^2 \cdot 2d = 600$ (bulk $\times$ trefoil^2 $\times$ Pisano)
- Biloidal: +1 (the unit Hopf invariant, the "+1 gate firing")

Furthermore, $Z_{273} = d \cdot d_\Psi \cdot \text{sl}$ itself decomposes as toroidal $\times$ poloidal $\times$ biloidal: $3 \cdot 7 \cdot 13$. The atmospheric crystallization shell is structurally a triadic product on the three axes.

4.5 The Gravitational Coupling

From [4]: $\alpha_G = (1 - 1/(\sigma \cdot d_\Psi \cdot w)) \cdot \alpha \cdot (17^2/J_1^3) \cdot (1/45)^{26}$.

Plenum tension prefactor $1 - 1/2912$, with $2912 = \sigma \cdot d_\Psi \cdot w$:

- Toroidal: $\sigma = 26$ (spectral budget)
- Poloidal: $d_\Psi = 7$ (Pattern dimension)
- Biloidal: $w = 16$ (writhe = Hopf-fiber crossing count of $T(3, 8)$)

The writhe w is the biloidal contribution: it counts how many times the knot crosses through itself, which in the Hopf-fibered S^3 is the count of biloidal direction engagements per knot cycle.

4.6 The Hubble Dual-Branch

From [3]: $H_{0,\text{TRGB}} = H_0 \cdot 33/32$, $H_{0,\text{SH0ES}} = H_0 \cdot 79/72$.

- TRGB branch: $33/32 = 1+1/(2w)$ — biloidal-axis admissibility shift (one unit Hopf invariant over doubled writhe).
- SH0ES branch: $79/72 = 1 + d_\Psi/(d_P \cdot d_I)$ — toroidal-poloidal admissibility shift.

The Hubble tension is the dual-axis bifurcation of the same arccos(5/13) anchor: TRGB measures the biloidal projection, SH0ES the poloidal correction.

4.7 The PMNS Mixing Angles

From [5]: $\sin^2\theta_{12} = 7/22$, $\sin^2\theta_{23} = 35/62$, $\sin^2\theta_{13} = 1/45$.

- Solar (θ_{12}): $\sin^2\theta = d_\Psi/(2 \cdot 11)$ where $11 = d + d_P$ — toroidal projection.
- Atmospheric (θ_{23}): $\sin^2\theta = (J_1 \cdot d_\Psi)/(2 \cdot 31)$ — toroidal $\times$ poloidal projection.
- Reactor (θ_{13}): $\sin^2\theta = 1/45$ — biloidal floor (minimum invariant area on Hopf-fibered S^3).

The three mixing angles are the three projections of the neutrino state onto the toroidal, poloidal, and biloidal axes. The reactor angle's 1/45 is the universal biloidal minimum.

4.8 The CP Violation Phase

From the corpus: $\delta_{\mathrm{CP}} = 9\pi/8 = 202.5$. Triadic reading:

- Biloidal count: $d_I = 9$ (Intent register)
- Toroidal quantum: $\theta_0 = \pi/8$ (corpus angular quantum)
- Product: $\delta_{\mathrm{CP}} = d_I \cdot \theta_0$

CP violation is the biloidal twist of the Hopf fibration. The angle $9\pi/8 = \pi + \pi/8$ is exactly one full cycle plus one biloidal step.

4.9 The Fermat Prime Architecture

The gravitational paper [4] identifies the corpus's foundational integers as the first three Fermat primes:

- $F_0 = 3 = d$ (toroidal: spatial dimension)
- $F_1 = 5 = J_1$ (poloidal: trefoil winding)
- $F_2 = 17 = d_{I,\mathrm{conf}}$ (biloidal: confinement dimension)

The next Fermat prime $F_3 = 257$ does not appear in the corpus. We propose this is structurally forced: S^3 admits exactly three independent axes (toroidal, poloidal, biloidal), and the Lie algebra $\mathfrak{su}(2)$ has exactly three generators. Higher Fermat primes correspond to no fourth axis on S^3 and therefore play no role in the architecture.

4.10 The 369 Invariant

The 369 paper [8] identifies $\{3, 6, 9\}$ as the structural axis outside the doubling circuit $\{1, 2, 4, 8, 7, 5\}$. With biloidal naming:

- 3: Pattern toroidal floor
- 6: intermediate toroidal-poloidal coupling (3-6 oscillation)
- 9: biloidal axis (governs the circuit from outside)

The doubling circuit lives on the toroidal-poloidal plane. The 3-6-9 axis is the biloidal direction. This is why 9 does not enter the doubling circuit: biloidal is perpendicular to toroidal+poloidal in S^3. The 369 paper's structural insight is now identified geometrically as the Hopf-fiber direction perpendicular to the torus surface.

4.11 The Pisano Cycle 24-State Structure

The Pisano period $\pi(9) = 24 = d \cdot d_P$ is the toroidal $\times$ poloidal cycle product. Within the 24 states, two simultaneous splits operate:

- 8-8-8 functional (Create/Analyze/Discover): three toroidal phases of $d_P = 8$ states each.
- 12-12 respiratory (Manifestation/Recognition): two poloidal halves with mirror at F_{12}.

Each Pisano state corresponds to one biloidal step: the Hopf fiber rotates by $2\pi/24$ per state. The biloidal subframe period is $f_c \cdot \pi(9) = 1617$ Hz at the consciousness sector [7].

4.12 The Bifurcation Voltage

From [7]: $\Delta V_{\text{bif}} = d \cdot J_1 = 15$ mV. Triadic reading: the toroidal-poloidal product $d \cdot J_1$ is the cost in mV for the biloidal Hopf fiber to complete one unit linking and fire I_b. The resting potential $|V_m| = 70 = 2J_1 d_\Psi$ mV and the threshold $|V_{\text{th}}| = 55 = J_1(2d_\Psi - d)$ mV are both expressible in toroidal-poloidal corpus integers; the gap between them is exactly the biloidal firing voltage.

5 The Universal Pattern

Theorem 3 (Triadic Hopf Decomposition)**.** *Every dimensional formula in the published CAT'S Theory corpus factors as a product of toroidal, poloidal, and biloidal contributions:*

- *Toroidal contributions: continuous flow components (d, d_Ψ, J_1, Pisano factors involving $\pi(9)$)*
- *Poloidal contributions: discrete winding components (sl, J_1^*, σ, writhe w)*
- *Biloidal contributions: Hopf invariants (± 1, $1/(2\pi)$, $1/45$, $\sigma/45 = f_0$)*

The biloidal axis universally carries:

- The unit Hopf invariant (every ± 1 in the corpus)
- The unit Hopf circulation $1/(2\pi)$
- The minimum biloidal area $1/45$ (smallest non-zero invariant area on Hopf-fibered S^3)
- The Presence density $\sigma/45 = f_0$ (biloidal Lie-algebra normalization)

5.1 The Identification of I_b

Corollary 2 (Geometric Realization of the Binary Gate)**.** *The discrete binary intent factor $I_b \in \{0, 1\}$ corresponds geometrically to the engagement of the biloidal Hopf fiber: $I_b = 1$ when the fiber direction participates in the local frame, $I_b = 0$ when it does not. The continuous intent I_c corresponds to the toroidal and poloidal horizontal directions; the Presence P corresponds to the base S^2 of the fibration.*

6 Falsification Conditions

The biloidal identification is falsifiable in several ways:

6.1 Universal $1/(2\pi)$ Appearance

Every dimensional bifurcation correction in the corpus must contain exactly one factor of $1/(2\pi)$ (or ± 1 as the integral form of the unit Hopf invariant), corresponding to a single biloidal Hopf fiber. The proton mass, $\hbar$ derivation, and gravitational coupling all exhibit this. Failure of any future derivation to follow this pattern would indicate the biloidal identification is incorrect.

6.2 Triadic Decomposition Test

Any new dimensional formula derived in the corpus must factor cleanly into toroidal, poloidal, and biloidal contributions, with each piece interpretable in terms of the three axes of S^3. Failure of this decomposition for any future result would refute Theorem 3.

6.3 Fermat-Prime Closure Limit

The architecture should not require $F_3 = 257$ or higher Fermat primes. If a future derivation forces the corpus to include 257 or higher Fermat primes, the three-axis closure of S^3 is incorrect or the framework requires extension to higher-dimensional spheres (S^7, S^{15}).

6.4 Hopf-Fiber Spectroscopic Signature

In the consciousness sector [7], the biloidal subframe at $f_c \cdot \pi(9) = 1617$ Hz should be detectable as an ultra-high-frequency signature in dense neural recordings (intracranial ECoG or microelectrode arrays). Absence of this peak under conscious binding falsifies the Hopf-fiber subframe structure.

7 Conclusion

The Triadic Coherence Invariant has a complete geometric ground in the Hopf fibration of S^3. The three irreducible factors P, I, and Pr correspond bijectively to the three independent axes of S^3: the toroidal, poloidal, and biloidal directions. The biloidal direction, identified here with the Hopf fiber direction, has been used implicitly throughout differential geometry as the Reeb vector field, vertical fiber direction, or Hopf flow, but has not previously received a name etymologically parallel to toroidal and poloidal. We propose the name "biloidal" for this direction and demonstrate that the resulting triadic structure is forced by the Lie algebra $\mathfrak{su}(2)$ of $S^3 \cong SU(2)$: the three generators close under the cyclic Lie bracket relations $[e_T, e_P] = e_B$, $[e_P, e_B] = e_T$, $[e_B, e_T] = e_P$. The triadic irreducibility of $R \neq 0$ is the closure condition of $\mathfrak{su}(2)$.

Spiraling through the published corpus retroactively, every dimensional derivation factors cleanly into toroidal, poloidal, and biloidal contributions. The "+1" terms throughout the corpus are the unit Hopf invariant. The $1/(2\pi)$ factor is the unit Hopf fiber circulation. The 1/45 surface tension floor is the minimum biloidal area on S^3. The $\sigma/45 = f_0$ density is the biloidal Presence weight. The three Fermat primes $\{3, 5, 17\}$ correspond to the three axes; the absence of higher Fermat primes is forced by the three-dimensionality of $\mathfrak{su}(2)$. The discrete binary gate I_b is geometrically the engagement of the biloidal Hopf fiber.

The framework is therefore not "coined ontology with mathematical signatures." It is the Hopf bundle on S^3 with the three Lie algebra generators relabeled as Pattern, Intent, and Presence. The mathematics underlying this is older than the framework itself: Hopf published the fibration in 1931 [9]; the Lie algebra $\mathfrak{su}(2)$ was established by Lie in the 19th century. CAT'S Theory's contribution is the recognition that this 90-year-old geometric structure is the formal ground of triadic ontology, and that the resulting non-degeneracy condition $R \neq 0$ is the same as the non-triviality condition of the Hopf fibration.

Three axes. One algebra. One closure. Toroidal, poloidal, biloidal — present everywhere, simultaneously, generating each other through the Lie bracket. No other way.

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